Method of Manufactured Solutions

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Let us consider the 2 dimensional Poisson's problem with manufactured solutions. Let Ω be a square domain. In this domain, the Poisson's equation would be the following.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x, y) = 0 \qquad\qquad \forall (x, y) \in \Omega \qquad\qquad (1)$$

In the method of manufactured solutions, we choose an analytical expression, let it be u_s, and plug it for $u(x, y)$ in equation 1 to manufacture the forcing function $f_m(x, y)$. Now we formulate the BVP to be solved numerically

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f_m(x, y) = 0 \qquad\qquad \forall (x, y) \in \Omega$$

$$u(x, y) = u_s(x, y) \qquad\qquad \forall (x, y) \in \partial\Omega$$

where $\partial\Omega$ is the boundary of the square domain. Let u_h be the function obtained by the numerical solver. Now, u_h and u_s can be compared to gauge the performance of the numerical solver.