Irodov Problem 1.13

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> **1.13** Point A moves uniformly with velocity v so that the vector $\vec{v}$ is continually 'aimed' at point B which in its turn moves rectilinearly and uniformly with velocity $u < v$. At the initial moment of time $\vec{v} \perp \vec{u}$ and the points are separated by a distance l. How soon will the points converge?

I want to discuss the solution of the above problem from 'Problems in General Physics' by I.E. Irodov [1]. Let T be the time taken for points A and B to converge. So, we need to find an expression for T in terms of the quantities u, v, and l.

1 School kid's solution

Let r be the distance between the points A and B, and θ be the angle between the velocities $\vec{v}$ and $\vec{u}$. We consider the relative velocity of A with respect to B along AB and orthogonal to AB to obtain the following relation.

$$\dot{r} = -v + u\cos\theta \tag{1}$$

$$r\dot{\theta} = -u\sin\theta \tag{2}$$

First, we will eliminate $\dot{\theta}$ by differentiating equation 1 with respect to time and then substituting the expression for $\dot{\theta}$ from equation 2.

$$\ddot{r} = -u\dot{\theta}\sin\theta$$

$$\implies r\ddot{r} = u^2\sin^2\theta$$

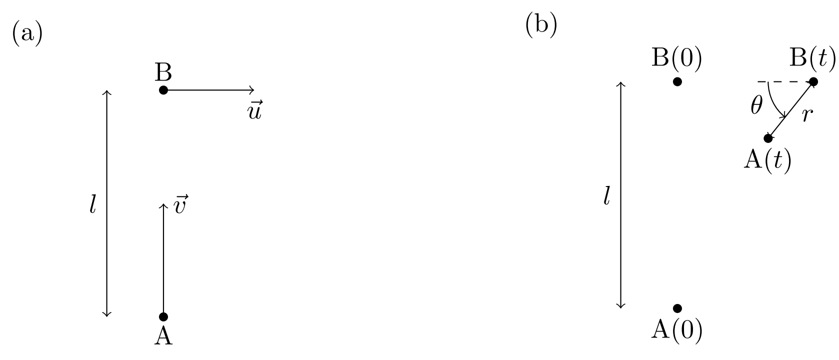

Figure 1: (a) Velocities of points A and B separated by a distance of l at the initial moment of time $t = 0$ (b) Velocities of points A and B separated by a distance of r at time $t \in (0, T)$.

Now, we will eliminate θ from the above expression by using the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$, and equation 1.

$$\begin{aligned} r\ddot{r} &= u^2 \sin^2\theta \\ &= u^2(1 - \cos^2\theta) \\ &= u^2\{1 - \left[\frac{\dot{r} + v}{u}\right]^2\} \end{aligned}$$

Simplifying the above expression we obtain the following second order differential equation.

$$\begin{aligned} r\ddot{r} + \dot{r}^2 + 2\dot{r}v + v^2 - u^2 = 0 \\ \implies \frac{d}{dt}(r\dot{r} + 2rv) + v^2 - u^2 = 0 \end{aligned}$$

Now, integrating with respect to time from $t = 0$ to T we get the following expression.

$$\begin{aligned} \int_{t=0}^{t=T} \frac{d}{dt}(r\dot{r} + 2rv)dt + \int_{t=0}^{t=T} (v^2 - u^2)dt = 0 \\ \implies r(T)\dot{r}(T) + 2r(T)v - r(0)\dot{r}(0) - 2r(0)v + (v^2 - u^2)T = 0 \end{aligned}$$

We will use the fact that

$$\begin{aligned} r(0) &= l \\ \dot{r}(0) &= -v + u\cos\pi/2 && \text{(from equation 1)} \\ &= -v \\ r(T) &= 0 && \text{(since A and B converge at time T)} \end{aligned}$$

to get the final expression for T.

$$T = \frac{lv}{v^2 - u^2} \tag{3}$$

2 Physics major's solution

To be written please wait

3 Numerical solution

To be written please wait

Discussion

To be written please wait

References

[1] I. E. Irodov, *Problems in General Physics*, Mir Publishers, 1988.